EE 232: Lightwave Devices
Lecture #19 – Semiconductor Laser modulation rate - Small signal analysis

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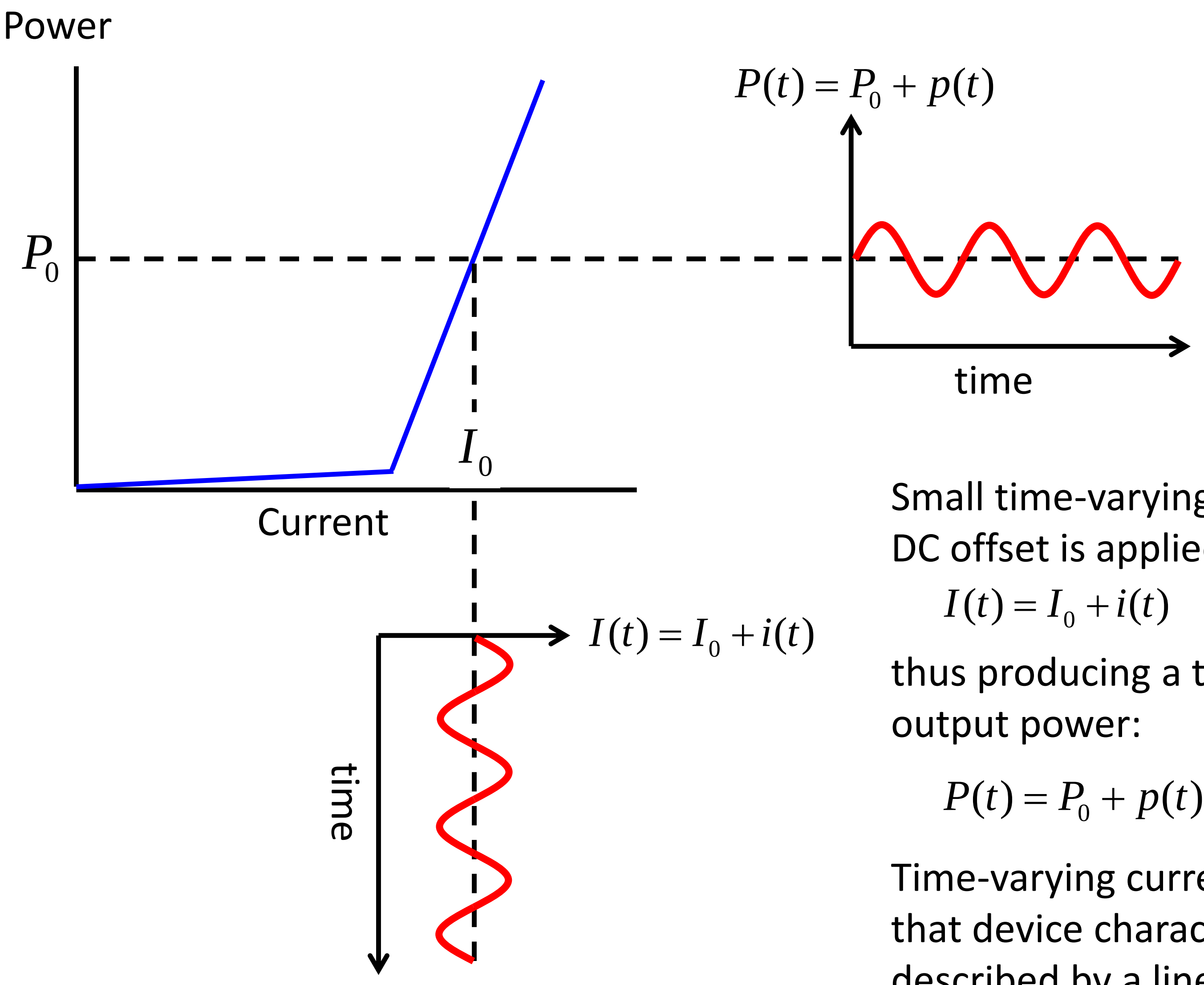
Small signal analysis



Small time-varying current with DC offset is applied to the device:

$$I(t) = I_0 + i(t)$$

thus producing a time dependent output power:

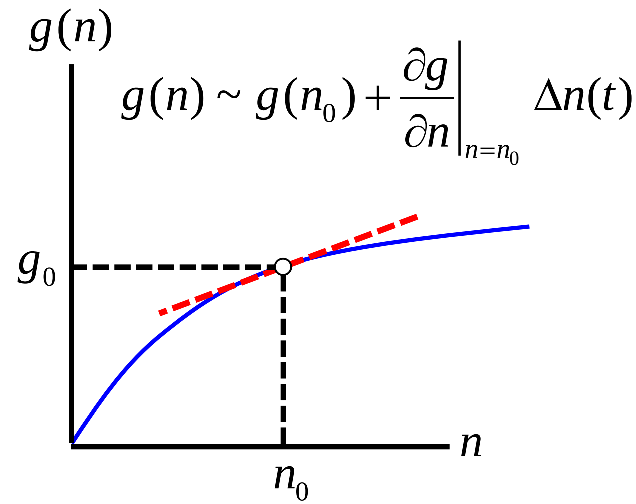
$$P(t) = P_0 + p(t)$$

Time-varying current is small such that device characteristic can be described by a linear extrapolation away from the bias point.

Small signal analysis - Laser

$$\frac{dn(t)}{dt} = \eta_i \frac{J(t)}{qd} - R(t) - v_g g(t) S(t) \qquad R(t) = R_{SRH}(t) + R_{sp}(t) + R_{Auger}(t)$$

$$\frac{dS(t)}{dt} = \Gamma \beta_{sp} R_{sp}(t) + \Gamma v_g g(t) S(t) - \frac{S(t)}{\tau_p}$$



Let $n(t) = n_0 + \Delta n(t)$

$J(t) = J_0 + \Delta J(t)$

$S(t) = S_0 + \Delta S(t)$

Then,

$$\frac{d}{dt}[n_0 + \Delta n(t)] = \eta_i \frac{J_0 + \Delta J(t)}{qd} - R_0 - \frac{\Delta n(t)}{\tau} - v_g[(g_0 + g'\Delta n(t))(S_0 + \Delta S(t))]$$

$$\boxed{\frac{d\Delta n(t)}{dt} = \eta_i \frac{\Delta J(t)}{qd} - \frac{\Delta n(t)}{\tau} - v_g[g_0 \Delta S(t) + g'\Delta n(t) S_0]}$$

(Ignoring 2nd harmonic term)

Similarly,

$$\boxed{\frac{d\Delta S(t)}{dt} = \Gamma \beta_{sp} \frac{\Delta n(t)}{\tau} + \Gamma v_g[g_0 \Delta S(t) + g'\Delta n(t) S_0] - \frac{\Delta S(t)}{\tau_p}}$$

Sinusoidal excitation – Laser

We assume the excitation is sinusoidal

$$\Delta n(t) = \mathrm{Re}[\Delta n(\omega) e^{-i\omega t}]$$

$$\Delta J(t) = \mathrm{Re}[\Delta J(\omega) e^{-i\omega t}]$$

$$\Delta S(t) = \mathrm{Re}[\Delta S(\omega) e^{-i\omega t}]$$

$$\Delta S(t)(-i\omega) \simeq \Gamma v_g [g_0 \Delta S + g' \Delta n S_0] - \frac{\Delta S(t)}{\tau_p}$$

Recall, $\frac{1}{\tau_p} = \Gamma v_g g_0 = \Gamma v_g g_{th}$

Then, $\Delta n = \frac{-i\omega}{\Gamma v_g g' S_0} \Delta S$

And,

$$\Delta n(-i\omega) = \eta_i \frac{\Delta J}{qd} - \frac{\Delta n(t)}{\tau} - v_g [g_0 \Delta S + g' \Delta n S_0]$$

$$\Delta n(-i\omega + \gamma) = \eta_i \frac{\Delta J}{qd} - v_g g_0 \Delta S$$

$$\frac{\Delta S}{\Gamma v_g g' S_0} (\omega_r^2 - \omega^2 - i\gamma\omega) = \eta_i \frac{\Delta J}{qd}$$

$$\boxed{\frac{\Delta S}{\Delta J} = \frac{\eta_i \Gamma v_g g' S_0 / qd}{\omega_r^2 - \omega^2 - i\gamma\omega}}$$

Note: $\gamma = \tau^{-1} + v_g g' S_0$ (damping)

$\omega_r^2 = \frac{v_g g' S_0}{\tau_p}$ (relaxation oscillation frequency)

Sinusoidal excitation – Laser (cont'd)

Let's write the transfer function in terms of power

$$\frac{\Delta P}{\Delta J} = \frac{\Delta S(h\nu v_g \alpha_m V_{act} / \Gamma)}{\Delta J} = \frac{\eta_i \Gamma v_g g' S_0 / qd}{\omega_r^2 - \omega^2 - i\gamma\omega}(h\nu v_g \alpha_m V_{act} / \Gamma)$$

$$\frac{\Delta P}{\Delta I} = \eta_i v_g \alpha_m \tau_p h\nu q^{-1} \left(1 - \frac{\omega^2}{\omega_r^2} - i\gamma \frac{\omega}{\omega_r^2}\right)^{-1}$$

$$= \eta_i \frac{h\nu}{q} \frac{\alpha_m}{\alpha_m + \alpha_i} \left(1 - \frac{\omega^2}{\omega_r^2} - i\gamma \frac{\omega}{\omega_r^2}\right)^{-1}$$

$$\boxed{\frac{\Delta P}{\Delta I} = \eta_i \frac{h\nu}{q} \frac{\alpha_m}{\alpha_m + \alpha_i} \left(1 - \frac{\omega^2}{\omega_r^2} - i \frac{\omega}{\omega_r}\left[\omega_r \tau_p + (\omega_r \tau)^{-1}\right]\right)^{-1}}$$

3dB-frequency

Electrical 3dB-frequency is given by

$$\left|1-\frac{\omega_{3dB}^{2}}{\omega_r^2}-i\frac{\omega_{3dB}}{\omega_r}\left[\omega_r\tau_p+(\omega_r\tau)^{-1}\right]\right|^{-1}=\frac{1}{\sqrt{2}}$$

It is often the case that $\omega_p\tau_p \ll 1$ and $\omega_r\tau \gg 1$. Then,

$$\left|1-\frac{\omega_{3dB}^{2}}{\omega_r^2}\right|^{-1}\simeq\frac{1}{\sqrt{2}}$$

$$f_{3dB}=\sqrt{1+\sqrt{2}}\,f_r$$

Writing in terms of output power,

$$f_{3dB}=\frac{1.55}{2\pi}\sqrt{\frac{v_g g' S_0}{\tau_p}}=\frac{1.55}{2\pi}\sqrt{\frac{v_g g'}{h\nu v_g\tau_p\alpha_m V_{cav}}}\sqrt{P_0}=\frac{1.55}{2\pi}\sqrt{\frac{v_g g'}{h\nu}\frac{1}{V_{cav}}\frac{\alpha_m+\alpha_i}{\alpha_m}}\sqrt{P_0}$$

Writing in terms of drive current,

$$f_{3dB}=\frac{1.55}{2\pi}\sqrt{\frac{v_g g'}{q}\frac{\eta_i}{V_{cav}}}\sqrt{I-I_{th}}$$

For high speed:

(1) Maximize differential gain
(2) Minimize cavity volume (mode volume)
(3) Maximize drive current relative to threshold current

Gain saturation

When the photon density is high, gain may decrease with further increase in photon density. This is called nonlinear gain saturation or gain compression. This can be accounted for with the following model.

$$g(n,S)=\frac{g(n_0)+g'(n(t)-n_0)}{1+\epsilon S(t)}$$

ϵ: gain compression factor

Let

$$n(t)=n_0+\Delta n(t)$$
$$J(t)=J_0+\Delta J(t)$$
$$S(t)=S_0+\Delta S(t)$$

$$g(n,S)\simeq\frac{g_0}{1+\epsilon S_0}+\Delta n(t)\left.\frac{\partial g}{dn}\right|_{n=n_0}+\Delta S(t)\left.\frac{\partial g}{dS}\right|_{n=n_0}$$

$$=\frac{g_0}{1+\epsilon S_0}+\frac{g'}{1+\epsilon S_0}\Delta n(t)-\frac{g_0}{(1+\epsilon S_0)^2}\epsilon\Delta S(t)$$

Then,

$$\frac{d}{dt}\begin{bmatrix}\Delta n(t)\\ \Delta S(t)\end{bmatrix}+\begin{bmatrix}A & D\\ -C & B\end{bmatrix}\begin{bmatrix}\Delta n(t)\\ \Delta S(t)\end{bmatrix}=\begin{bmatrix}\eta_i\Delta J(t)/qd\\ 0\end{bmatrix}$$

$$A=\frac{1}{\tau}+\frac{v_g g' S_0}{1+\epsilon S_0}\qquad B=\frac{1}{\tau_p}-\frac{\Gamma v_g g_0}{(1+\epsilon S_0)^2}\qquad C=\frac{\Gamma v_g g' S_0}{1+\epsilon S_0}\qquad D=\frac{v_g g_0}{(1+\epsilon S_0)^2}$$

Sinusoidal excitation with gain saturation

Let $\Delta n(t) = \mathrm{Re}[\Delta n(\omega) e^{-i\omega t}]$

$\Delta J(t) = \mathrm{Re}[\Delta J(\omega) e^{-i\omega t}]$

$\Delta S(t) = \mathrm{Re}[\Delta S(\omega) e^{-i\omega t}]$

$$\begin{bmatrix} -i\omega + A & D \\ -C & -i\omega + B \end{bmatrix} \begin{bmatrix} \Delta n(\omega) \\ \Delta S(\omega) \end{bmatrix} = \begin{bmatrix} \eta_i \Delta J(\omega) / qd \\ 0 \end{bmatrix}$$

$$\Delta S = \frac{C\eta_i \Delta J / qd}{(AB + CD) - \omega^2 - i(A+B)\omega}$$

$$\omega_r^2 = AB + CD$$

$$= \frac{1}{\tau_p} \frac{v_g g' S_0}{1 + \epsilon S_0} - \frac{1}{\tau}\left[\frac{1}{\tau_p} - \frac{\Gamma v_g g_0}{(1+\epsilon S_0)^2}\right]$$

$$\boxed{\omega_r^2 \simeq \frac{1}{\tau_p} \frac{v_g g' S_0}{1 + \epsilon S_0}}$$

$$\gamma = A + B$$

$$= \frac{1}{\tau} + \frac{v_g g' S_0}{1 + \epsilon S_0}\left(1 + \frac{\epsilon}{v_g g' \tau_p}\right) + \beta \frac{R_{sp}}{S_0 (1 + \epsilon S_0)}$$

$$\boxed{\gamma \simeq \tau^{-1} + K f_r^2 \qquad K = 4\pi^2 \left(\tau_p + \frac{\epsilon}{v_g g'}\right)}$$

Sinusoidal excitation with gain saturation

Then,

$$\frac{\Delta S}{\Delta J} = \frac{\Gamma v_g g' S_0}{1+\epsilon S_0} \eta_i (qd)^{-1} \left(\omega_r^2 - \omega^2 - i\gamma\omega\right)^{-1}$$

$$\frac{\Delta P}{\Delta J} = \frac{\Delta S}{\Delta J}(h\nu v_g \alpha_m V_{act} / \Gamma) = \frac{v_g g' S_0}{1+\epsilon S_0} \eta_i h\nu v_g \alpha_m V_{act} (qd)^{-1} \left(\omega_r^2 - \omega^2 - i\gamma\omega\right)^{-1}$$

$$\frac{\Delta P}{\Delta I} = \frac{v_g g' S_0}{1+\epsilon S_0} \eta_i h\nu v_g \alpha_m q^{-1} \left(\omega_r^2 - \omega^2 - i\gamma\omega\right)^{-1}$$

$$= \eta_i h\nu v_g \tau_p \alpha_m q^{-1} \left(1 - \frac{\omega^2}{\omega_r^2} - i\gamma \frac{\omega}{\omega_r^2}\right)^{-1}$$

$$= \eta_i \frac{h\nu}{q} \frac{\alpha_m}{\alpha_m + \alpha_i} \left(1 - \frac{\omega^2}{\omega_r^2} - i\gamma \frac{\omega}{\omega_r^2}\right)^{-1}$$

$$\boxed{\frac{\Delta P}{\Delta I} = \eta_i \frac{h\nu}{q} \frac{\alpha_m}{\alpha_m + \alpha_i} \left(1 - \frac{\omega^2}{\omega_r^2} - i \frac{\omega}{\omega_r}\left[(2\pi)^{-2} K \omega_r + (\omega_r \tau)^{-1}\right]\right)^{-1}}$$

K-factor

K : Referred to as the "K factor". Units are seconds. As we will see, the K factor sets the upper limit of the intrinsic laser modulation speed.

Comparison with our previous result where gain saturation was not considered

With gain saturation

$$\gamma = \tau^{-1} + \frac{K}{4\pi^2}\omega_r^2$$

Without gain saturation

$$\gamma = \tau^{-1} + \tau_p \omega_r^2$$

Typical K-factor is about 100 times larger than typical photon lifetime (Coldren pg. 260). Therefore, as drive current is increased and the relaxation oscillation frequency is increased, the damping factor becomes non-negligible. Previously, we ignored the damping to calculate the 3dB frequency; however, this is not accurate at high photon densities (i.e. high drive current) in the presence of gain saturation.

3dB-frequency

$$\frac{\Delta P}{\Delta I} = \eta_i \frac{h\nu}{q} \frac{\alpha_m}{\alpha_m + \alpha_i} \left(1 - \frac{\omega^2}{\omega_r^2} - i \frac{\omega}{\omega_r} \left[(2\pi)^{-2} K \omega_r + (\omega_r \tau)^{-1} \right] \right)^{-1}$$

Let's try to find the maximum 3dB-frequency that is possible

$$\left| 1 - \frac{\omega_{3dB}^2}{\omega_r^2} - i \frac{\omega_{3dB}}{\omega_r^2} \gamma \right|^{-1} = \frac{1}{\sqrt{2}}$$

$$\left[\left(1 - \frac{\omega_{3dB}^2}{\omega_r^2} \right)^2 + \left(\frac{\omega_{3dB}}{\omega_r^2} \gamma \right)^2 \right]^{-1} = \frac{1}{2}$$

We can see that the 3dB frequency is maximized when $1 - \frac{\omega_{3dB}^2}{\omega_r^2} = 0$

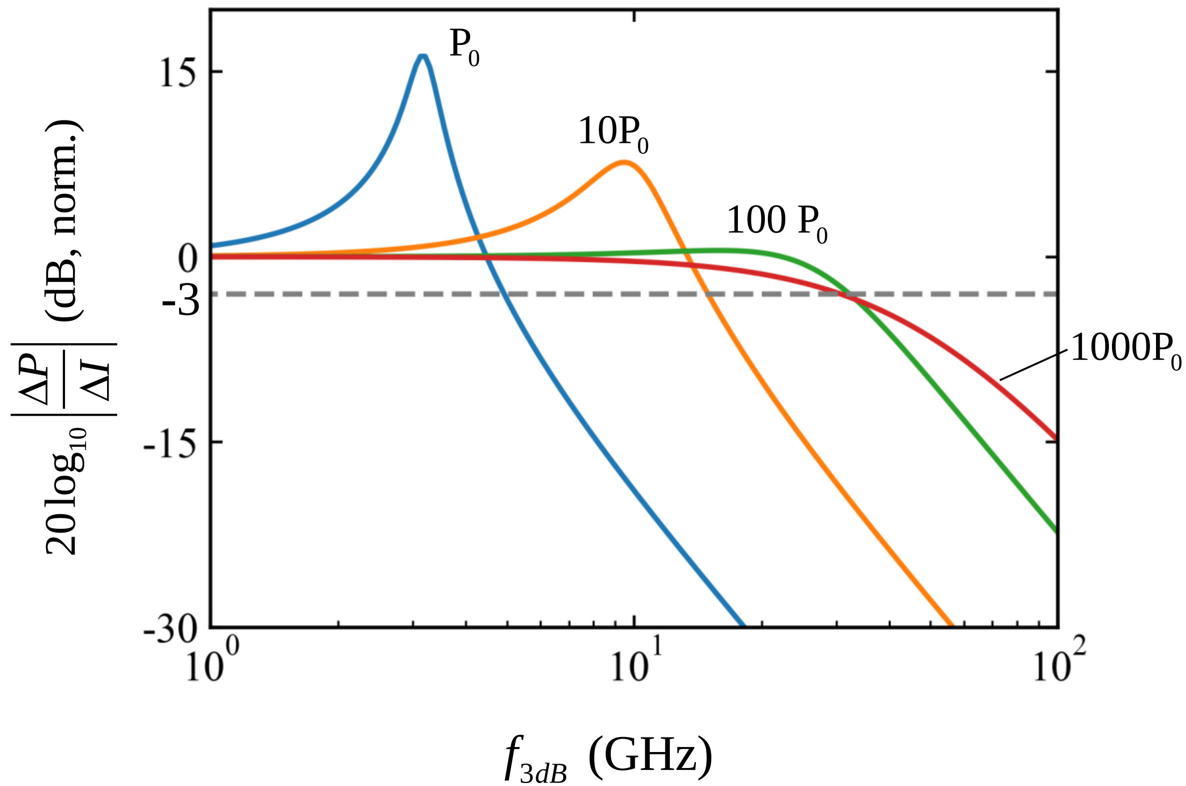
Then,

$$\left[\left(\frac{\omega_{3dB,\max}}{\omega_r^2} \gamma \right)^2 \right]^{-1} = \frac{1}{2}$$

$$\omega_{3dB,\max} = \sqrt{2}\omega_r^2 \gamma^{-1} \rightarrow \boxed{f_{3dB,max} \simeq \frac{2\pi\sqrt{2}}{K}}$$

K-factor is an intrinsic parameter that sets the upper limit of the modulation speed

3dB-frequency



At low power (low photon density): Damping is small. 3dB frequency is increased by increasing the relaxation oscillation frequency through increased current injection.

At high power (high photon density): Damping is large. Relaxation oscillation frequency saturates due to gain compression. Maximum 3dB frequency is limited by K-factor.